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Technical Note

On oscillatory Marangoni convection in rotating fluid layer with flat free surface subject to uniform heat flux from below

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Abstract

The effect of uniform rotation on the onset of steady and oscillatory surface-tension-driven (Marangoni) convection in a horizontal fluid layer with a flat free upper surface and heated from below with a uniform heat flux is considered theoretically using linear stability theory. The *Ta–Pr* parameter space, divided into domains in which either steady or oscillatory convection is preferred, is presented.

Keywords: Marangoni convection; Rotation; Oscillatory

1. Introduction

The problem of surface-tension-driven (Marangoni) convection in a fluid layer has been the subject of a great deal of theoretical and experimental investigation since the pioneering theoretical work of Pearson [1]. In his pioneering linear stability theory, Pearson [1] considered both the so-called "conducting" case of a constant temperature rigid lower boundary of the horizontal fluid layer at which no perturbation in temperature is allowed and the so-called "insulating" case of a constant heat flux lower boundary at which no perturbation in the heat flux is allowed. The insulating case is realizable if the lower boundary is very thin and has a controlled heat flux [2]. Vidal and Acrivos [3] and Takashima [4] showed numerically that oscillatory Marangoni convection is impossible when the free surface is non-deformable. This conclusion has been confirmed analytically by Vrentas and Vrentas [5].

Vidal and Acrivos [6] extended Pearson's [1] work to include the effect of rotation. Their analysis was further extended to include other aspects of the problem by McConaghy and Finlayson [7], Namikawa et al. [8], Takashima and Namikawa [9] and Sarma [10]. Kaddame and Lebon [11,12] obtained the critical points for the onset of steady and oscillatory Marangoni convection with rotation for the conducting case when the Marangoni number, $M > 0$. While Char et al. [13] and Chang and Chiang [14] studied the onset of oscillatory convection in the $M < 0$ regimes only. Very recently, Hashim and Sarma [15] divided the *Ta–Pr* (where *Ta–Pr* are the Taylor number and Prandtl number) parameter space into domains in which either steady or oscillatory Marangoni convection is preferred for the conducting case when $M > 0$. In this work, we shall demonstrate that altering the thermal boundary condition at the lower surface has quite a significant effect on the *Ta–Pr* parameter space as compared to the conducting case. Thus the investigation for the insulating case is not without interest.

2. Mathematical model

We wish to examine the stability of a horizontal layer of quiescent fluid of thickness d which is unbounded in the horizontal x- and y-directions. The layer is kept rotating uniformly around a vertical axis with a constant angular velocity Ω.

Following the classical lines of linear stability theory as presented by Chandrasekhar [16], the linearised and dimensionless governing equations can be written as,

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Nomenclature

a	total horizontal wave number
d	initial thickness of the layer
g	gravitational acceleration
k	thermal conductivity
M	Marangoni number
Ta	Taylor number
Pr	Prandtl number
s	temporal growth rate
$\theta(z)$	vertical variation of temperature perturbation
$W(z)$	vertical variation of vertical velocity perturbation
$K(z)$	vertical variation of vertical vorticity perturbation
x, y, z	spatial cartesian coordinates
Greek symbols	
γ	coefficient of surface tension
κ	thermal diffusivity
ν	kinematic viscosity
ρ	density of fluid
σ	growth rate
ω	frequency
Ω	angular velocity
ΔT	temperature difference
Subscript	
c	critical state

$$(D^2 - a^2)(D^2 - a^2 - s)W = Ta^* DK, \tag{1}$$

$$(D^2 - a^2 - s)K = -DW, \tag{2}$$

$$(D^2 - a^2 - sPr)\Theta = -W, \tag{3}$$

where W, Θ and K are amplitudes of vertical velocity, temperature and vertical vorticity, respectively. The equations have been written in dimensionless form using d/π, $d^2/\pi^2\nu$ and $\nu\Delta T/\pi\kappa$ as the scales for distance, time and temperature, respectively, where ν is the kinematic viscosity and κ is the thermal diffusivity. The operator $D = \mathrm{d}/\mathrm{d}z$ denotes differentiation with respect to the vertical coordinate z, $a = kd/\pi$ is the dimensionless wavenumber and $s = \tilde{s}d^2/\pi^2\nu$ is the stability parameter, where k is the thermal conductivity of the fluid. The stability parameter s is in general a complex variable denoted by $s = \sigma + \mathrm{i}\omega$, where σ is the growth rate of the instability and ω is the frequency. If $\sigma > 0$, the disturbances grow and the system becomes unstable. If $\sigma < 0$, the disturbances decay and the system becomes stable. When $\sigma = 0$, the instability of the system, at the marginal state, sets in as steady motion, provided $\omega = 0$, or as oscillatory motion, provided $\omega \neq 0$.

Let us now assume that the layer is bounded below by a rigid boundary, which is kept at a uniform heat flux, and above by a perfectly insulated, flat free surface, subject to a uniform vertical temperature gradient. The boundary conditions are then given by

$$W = 0, \tag{4}$$

$$(D^2 + a^2)W + a^2 M^* \Theta = 0, \tag{5}$$

$$D\Theta = 0, \tag{6}$$

$$DK = 0, \tag{7}$$

on $z = \pi$, and

$$W = DW = D\Theta = K = 0, \tag{8}$$

on $z = 0$. The starred dimensionless numbers are defined by $M^* = M/\pi^2$ and $Ta^* = Ta/\pi^4$, where the Marangoni number, $M = \gamma\Delta T d/\rho_0\nu\kappa$, the Taylor number, $Ta = 4\Omega^2 d^4/\nu^2$, and the Prandtl number, $Pr = \nu/\kappa$. Here γ is the coefficient of surface tension, ρ is the density and ΔT is the temperature difference between the top and bottom surfaces.

3. Solution approach

Since the solution method is standard, we only give a brief description of the solution approach in this section. Following similar procedure as employed by Hashim and Sarma [15], the homogeneous boundary-value problem (1)–(3) subject to boundary conditions (8) can be turned into the eigenvalue problem of the form

$$f(M, Ta, Pr, a, s) = 0, \tag{9}$$

from which the stability domains of the problem can be obtained. The real and imaginary parts of (9) give a system of two nonlinear equations for the eigenvalues s and M. The NAG Fortran routine C05NBF is then used, given values for the other parameters, to find a zero, $\omega = \mathrm{Im}(s(Ta, Pr, a))$ and $M = M(Ta, Pr, a)$ with $\sigma = 0$, of the system of nonlinear equations.

4. Results and discussion

The most relevant parameters of the current problem are the Taylor number Ta and the Prandtl number Pr. It has been shown that the effect of increasing Ta is to inhibit the onset of convection (cf. [6,16,13,14]). As in the conducting case studied by McConaghy and Finlayson [7], oscillatory convection was also possible in the insulating case. The influence of Pr on the existence of oscillatory convection is important, since increasing viscosity would suppress convection. On the basis of the complete marginal stability curves, there exists Pr_c such that for all $Pr < Pr_c$, the critical Marangoni number M_c for oscillatory convection is always less than the critical number for steady convection.

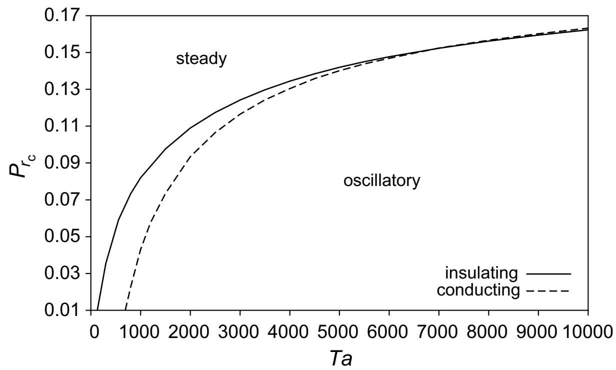

Fig. 1. Pr_c below which oscillatory convection is preferred plotted as functions of Ta.

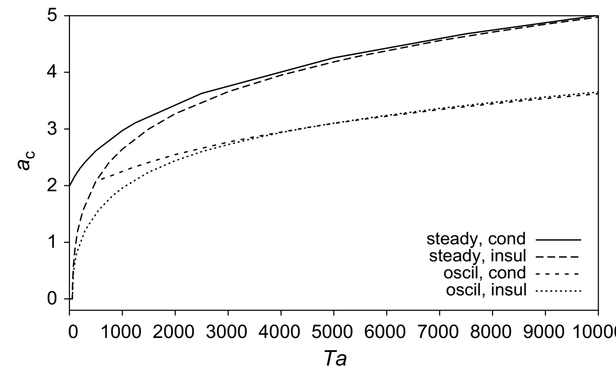

Fig. 3. a_c plotted as functions of Ta where Pr is given by the corresponding Pr_c in Fig. 1.

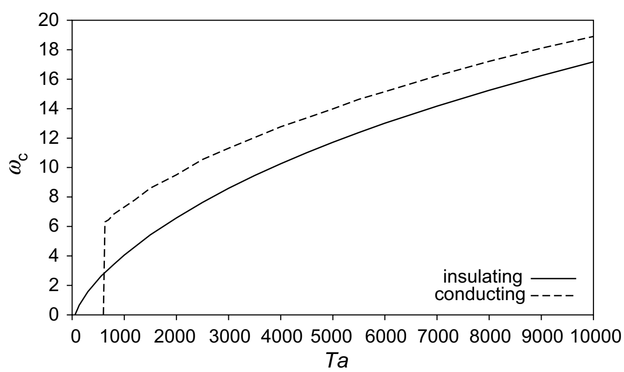

Fig. 2. ω_c plotted as functions of Ta where Pr is given by the corresponding Pr_c in Fig. 1.

In Figs. 1 and 2 we plot Pr_c and ω_c as functions of Ta, respectively. The curve in Fig. 1 defines the boundary between the steady and oscillatory domains, i.e. points below the curve represent parameter combinations (Ta, Pr_c) for which convection sets in as oscillatory motions, while points above the curve are those for which steady convection is preferred. As Ta increases, the region for oscillatory convection widens. The result corresponding to the conducting case obtained by Hashim and Sarma [15] is also shown on the same figure for direct comparison. Note that the region of oscillatory convection for the insulating case is larger than that for the conducting case when Ta is less than the point $Ta = Ta^*$ at which the two curves crisscross. When $Ta > Ta^*$ the reverse is true. The critical frequencies for the insulating case are lower than that for the conducting case as depicted in Fig. 2 and the ω_c for the conducting case jumps from zero to a non-zero value much later than that of the insulating case as Ta increases. This corresponds to a sudden changeover from convection setting in as steady motions to that setting in as oscillatory motions. Fig. 3 shows the critical wavenumber at the onset of steady and oscillatory convection for both the insulating and conduction cases. We note that, in particular, a_c for the insulating case is smaller than that of the conducting case when Ta is less than the point $Ta = Ta^*$ at which the two curves crisscross. However, when $Ta > Ta^*$ the reverse is true, which is quite unexpected.

5. Conclusion

In this work, we have determined how the Ta–Pr parameter space is divided into domains in which either steady or oscillatory Marangoni convection is preferred. Oscillatory convection is possible for fluids of very small Prandtl number, Pr. Liquid metals including mercury fall into this group of fluids with Pr in the range 0.004–0.015. The theoretical prediction in this paper is hoped to be a useful guide for the experimentalists. A detailed discussion on a more general configuration will constitute the subject of our subsequent investigation.

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